Proposition 1: Let $f$ and $g$ be nonnegative measurable functions defined on a measurable set $E$. If $f = g$ a.e., then
\[ \int_E f = \int_E g. \]

Proposition 2: Let $f$ be a nonnegative measurable function defined on a measurable set $E$. Let $A$ and $B$ be measurable subsets of $E$. Suppose that $f$ vanishes off of $A$ and that $f$ also vanishes off of $B$. Then
\[ \int_A f = \int_B f. \]

Notation: Let $f$ be a nonnegative measurable function defined on a measurable set $E$, and suppose $f$ vanishes off of a measurable set $A \subset E$. We let $f|_A$ denote $\int_A f$.

Proposition 3: Let $f$ be a nonnegative measurable function defined on a measurable set $E$. Let a measurable set $A \subset E$ be given. Then
\[ \int_A f = \int f \chi_A. \]

Proposition 4: Let $f$ be a nonnegative measurable function defined on a measurable set $E$. Let $A$ and $B$ be measurable subsets of $E$, and suppose $A \cap B = \emptyset$. Then
\[ \int_{A \cup B} f = \int_A f + \int_B f. \]

Proposition 5: Let $f$ be a nonnegative measurable function defined on a measurable set $E$. Let $A$ be a measurable subset of $E$. If $mA = 0$, then
\[ \int_A f = 0. \]

Proposition 6: Let $f$ be a nonnegative measurable function defined on a measurable set $E$. Let $A$ and $B$ be measurable subsets of $E$, and suppose $A \subset B$. If $m(B \sim A) = 0$, then
\[ \int_A f = \int_B f. \]

Proposition 7: Let $E$ be a measurable subset of $\mathbb{R}$. Then $\int \chi_E = mE$. This is true whether $mE$ is finite or infinite.

Proposition 8: Let $\langle a_n \rangle_{n=1}^{\infty}$ be a sequence of extended real numbers. Suppose that $\ell$ is an extended real number and that $\lim a_n = \lim a_n = \ell$. Then $\lim a_n = \ell$.

Proposition 9: Let $f$ be a nonnegative measurable function on a measurable set $E$. Then
\[ \int_E f < \infty \Rightarrow m\{x : f(x) = \infty\} = 0. \]