Homework: Apply the forgoing analysis to the "hyperbolic-elliptic functions," defined by

$$\frac{x^2}{a^2} - y^2 = 1,$$

let

$$\mathrm{sh}(u,k) \;=\; y, \quad \mathrm{ch}(u,k) \;=\; \frac{x}{a},$$

and

$$\mathrm{dh}(u,k) \;=\; \frac{r}{a}$$

Derive their properties. You should find for instance

$$\mathrm{ch}^2(u,k) - \mathrm{sh}^2(u,k) \;=\; 1,$$

$$\frac{d}{du}\mathrm{sh}(u,k) \;=\; \mathrm{ch}(u,k)\mathrm{dh}(u,k),$$

$$\frac{d}{du}\mathrm{ch}(u,k) \;=\; \mathrm{sh}(u,k)\mathrm{dh}(u,k).$$