Lesson 0: Useful Information!

I. You can find a file of updates and corrections for the syllabus at my web site:

http://www.und.edu/instruct/metzger

If you find errors in the syllabus not mentioned there, please let me know.

My e-mail address is jerry.metzger@und.edu

II. If you get stuck on a problem, I'm happy to answer questions via e-mail at the address above. Phone calls and office visits generally are not convenient for me. If you have a question about an assignment, be sure to specify the course number, chapter, section, and problem number in your e-mail. Describe how far you can get with the problem, and what part has you stumped. While I am willing to respond to questions concerning general methods, and to provide hints when you are stuck, I am not willing to answer questions of the *did I get the right answer* variety.

III. No more that three assignments may be submitted in a one week period. This rule is in place to guarantee sufficient time is devoted to the material and to give me sufficient time to grade the assignments for everyone.

IV. For the problems,you have to show the work to receive credit for a solution, at least for problems that require more than one step.

V. If you have access to a computer and the internet, you might want to consider submitting your assignments directly through Blackboard as attachments. Details for attaching your homework problems are on the lesson page in Blackboard. Assignments submitted via Blackboard will normally be graded and corrections or comments uploaded to Blackboard within two to three days. The other option is to mail paper versions of the written assignments to the Department of Correspondence & Online Study. They will be graded and returned to you. A turn-around time of 15 days or so seems to be typical for assignments sent via the post office. All grades will be posted in Blackboard once assignments are graded.

To submit assignments via the Blackboard system, save your work to a file, and attach the file via the View/Complete Assignment link.

The following are some options for submitting assignments via Blackboard:

1: For each lesson, the assigned problems must be submitted as a single .pdf file. Info about .pdf format, and about combining separate files into a single file are given at the end of this document.

2. Probably the most convenient option is to write solutions out on paper, then scan or take a digital picture of the document. Save the document as a .pdf file (the best choice if available) or a .gif file, or some other common graphics format. Convert non-pdf files to .pdf format, and combine files into a single page.

3. If you want to mail paper versions of the assignments, send the work to

Department of Correspondence & Online Studies

Division of Continuing Education

University of North Dakota

Gustafson Hall Room 103

3264 Campus Road Stop 9021

Grand Forks, ND 58202-9021

VI. When giving numerical answers to problems:

a) Always write exact answers when possible rather than decimal approximations. For example, write $x = \sqrt{2}$ rather than $x = 1.414$, and write $x = 2\pi$ rather than $x = 6.2832$.

b) Write fractions rather than mixed numbers. For example, write $\frac{17}{5}$ rather than $3\,\frac{2}{5}$.

VII. There are some common mathematical errors you need to watch out for. The following list is organized roughly by course level. The first part of the list lists errors related to material in Intermediate Algebra. The end of the list discusses errors related to Calculus.

There are many site on the web where these, and related errors, are discussed. Two that particularly complete are:

1) http://tutorial.math.lamar.edu/AllBrowsers/CommonErrors/CommonMathErrors.asp

and

2) http://www.math.vanderbilt.edu/ schectex/commerrs

Common Mathematical Errors

Mathematics is a language. Like all languages it has its rules of grammar that permit clear, efficient, and succinct communication. Part of learning mathematics is learning good grammar. There's no reason you should have an inborn understanding of this grammar. It is something you have to learn, mostly through practice and imitation. Eventually, bad mathematical grammar will be as offensive to your eye as the sentence *Thems gots no cash!* is to your ear. Some of the errors below fall into this mathematical grammar classification. Others are arithmetic errors committed when the rules of algebra are applied incorrectly. Finally, some other errors are of scholarship such as not checking to see if proposed solutions are reasonable.

1) **division by 0**: The expression $\frac{7}{0}$ has no meaning. In fact, no fraction with denominator equal to 0 has a meaning. Division by 0 is undefined. There is no logically consistent way to assign a value to expressions such as $\frac{7}{0}$. In particular, the following are all wrong:

WRONG! $\frac{7}{0} = 0$ **WRONG!**

WRONG! $\frac{7}{0} = 7$ **WRONG!**

WRONG! $\frac{7}{0} = \infty$ **WRONG!**

So if you see an expression such as $\frac{7}{0}$ you should write something like: $\frac{7}{0}$ *is undefined.*
A picky point: Don't write $\frac{7}{0}$ = *undefined.* When something is undefined, it doesn't equal anything! So writing = there is a mathematical grammar error.

2) **unneeded coefficients**: The expression $1x$ means 1 *times* x. The 1 is called a coefficient in that expression. And since 1 times anything is just that thing again, the 1 is completely unnecessary. While strictly speaking it isn't wrong to write the coefficient 1, mathematical grammar insists that it be omitted. So if you are tempted to write $1x$, just write x instead. Likewise, the expression x^1 just means x, so omit that exponent of 1. Of course, all this goes for other expressions in place of x. So, for example, write $a^2 + b$ instead of $1(a^2 + b)$ or $a^2 + 1b$, and write $2 + xy$ instead of $(2 + xy)^1$.

3) **minus isn't negative**: If a represents a real number, you cannot be sure whether $-a$ is positive or negative. For example, if $a = 5$, then $-a = -5$, definitely a negative number. But if $a = -7$, then $-a = -(-7) = 7$, a positive number. Also, if $a = 0$, then $-a = 0$, neither positive nor

negative. In an algebraic expression, don't confuse the plus and minus signs with the concepts of positive and negative.

4) **what is x?**: When introducing variables in a problem, be sure to make it clear what the variable represents. For example, write *Let x be the speed of the first car*, or *Let t be the time the large pipe needs to empty the pool.* The meaning of a variable can also be indicated in some cases by drawing a diagram and using the variable as a label. For example, if you draw a rectangle, and write x near one side and y near and adjacent side, that is good enough to tell the reader that x and y represent the side lengths of the rectangle. The English equivalent of forgetting to tell the reader what the variable represents would be to have someone, out of the blue, say *It is two years old.* That would surely lead you to ask *What is it?*. When x appears out of the blue in the solution to a problem, the natural question is *What is x?*.

5) **keeping it together**: Parentheses are used to group items together so that the way the quantities are to be combined is not ambiguous. Below are a few examples to show the correct way to interpret parentheses. Each example consists of a pair of expressions that can be easily confused.

$$2(5+4) = 2 \cdot 9 = 18 \qquad 2 \cdot 5 + 4 = 10 + 4 = 14$$

$$-6^2 = -6 \cdot 6 = -36 \qquad (-6)^2 = (-6) \cdot (-6) = 36$$

$$(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd \qquad a + b(c+d) = a + bc + bd$$

$$a + b \cdot c + d = a + (bc) + d \qquad (a+b) \cdot (c+d) = ac + ad + bc + bd$$

$$(2x)^3 = 2^3 x^3 = 8x^3 \qquad 2x^3 = 2 \cdot x^3 (not\ 8x^3)$$

$$(2+3)/(6+7) = 5/13 \qquad 2 + 3/6 + 7 = 2 + (1/2) + 7 = 9 + (1/2) = 19/2$$

6) **write right**: Careless writing can change the meaning of an expression. Here are three examples of careless penmanship:

(1) writing x to the power $\frac{2}{3}$ as $x\frac{2}{3}$ when it should be $x^{\frac{2}{3}}$. The wrong version would be interpreted as x times $\frac{2}{3}$.

(2) writing the cube root of x as ${}^3\sqrt{x}$ instead of $\sqrt[3]{x}$. The wrong version would be interpreted as 3 times the square root of x.

(3) writing $\dfrac{2}{3x}$ or $2/3x$ instead of $\dfrac{2x}{3}$ or $\left(\dfrac{2}{3}\right)x$. The wrong versions would be interpreted as 2 divided by the quantity $3x$, probably not what was intended.

7) **squaring and rooting**: The tempting looking formulas below are both wrong:

$$\textbf{WRONG!} \qquad (a+b)^2 = a^2 + b^2 \qquad \textbf{WRONG!}$$

$$\textbf{WRONG!} \qquad \sqrt{a+b} = \sqrt{a} + \sqrt{b} \qquad \textbf{WRONG!}$$

To see these are not correct, think about the following two examples:

$$(2+3)^2 = 5^2 = 25 \qquad \text{but} \qquad 2^2 + 3^2 = 4 + 9 = 13$$

$$\sqrt{9+16} = \sqrt{25} = 5 \qquad \text{but} \qquad \sqrt{9} + \sqrt{16} = 3 + 4 = 7$$

8) **signs make bad neighbors**: The arithmetic signs (addition, subtraction, multiplication, and (the only rarely used in real life) division sign $\div$) cannot be written directly adjacent to each other. As examples, the following are all wrong:

$$\textbf{WRONG!} \qquad a++b \qquad a+-b \qquad a-+b \qquad a--b \qquad \textbf{WRONG!}$$

There are two ways to correct the error. Either use parentheses:

$$a+(+b) \qquad a+(-b) \qquad a-(+b) \qquad a-(-b)$$

or else simply write the one correct sign:

$$a+b \qquad a-b \qquad a-b \qquad a+b$$

9) <u>**square roots can't be negative**</u>: The symbol $\sqrt{x}$ stands for the number 0 or bigger whose square is x. For example $\sqrt{25} = 5$. Not

$$\textbf{WRONG!} \qquad \sqrt{25} = \pm 5 \qquad \textbf{WRONG!}$$

Notice that $-\sqrt{25} = -5$.

This error probably comes from confusing $\sqrt{25}$ with the problem of solving the equation $x^2 = 25$. There are two solutions to this equation: $x = 5$ and $x = -5$. Another way to say that: The solutions to $x^2 = 25$ are $x = \pm\sqrt{25} = \pm 5$.

10) <u>**= doesn't mean so**</u>: The following is not a correct solution of the equation $2x + 5 = 11$.

$$\textbf{WRONG!} \qquad 2x + 5 = 11 = 2x = 6 = x = 3 \quad \textbf{WRONG!}$$

The error being made here is the use of the equal sign as a replacement for the word *so*. Notice that included in the *solution* above are such nonsensical statements as $11 = 6$ and $6 = 3$. A correct way to write the solution would be to actually write in *so* where it belongs:

$$2x + 5 = 11 \qquad \text{so} \qquad 2x = 6 \qquad \text{so} \quad x = 3$$

An alternative is to write each new fact on a new line. It is understood that each new line begins with an unwritten *so*. The solution would look like:

$$2x + 5 = 11$$
$$2x = 6$$
$$x = 3$$

A good rule of thumb is: **Only write = between expressions that are equal!**.

11) <u>**factors are not terms**</u>: When two expressions are multiplied together, each expression is called a **factor**. When two expressions are added, each expression is called a **term**. For example, xy has two factors, x and y. We can think of $(a + b)^2(c + 2)$ as having two factors also: $(a + b)^2$ and $(c + d)$. The concept of is a little bit vague. For example, in $(a + b)^2(c + d)$ we might also

justifiably say there are three factors: $(a+b)$, another $(a+b)$, and $(c+d)$. But the point is a factor is one of a list of things being multiplied together.

Now think about $x+2y+17z$ This expression is made up of three terms: x, $2y$, and $17z$. Likewise $xy^2+(a+b)^3$ is made up of two terms: xy^2 and $(a+b)^3$,

The reason there is so much fuss about factors and terms is that when reducing fractions, common factors can be canceled, common terms cannot. For example:

$$\textbf{WRONG!} \qquad \frac{a+b}{a+c} = \frac{b}{c} \qquad \textbf{WRONG!}$$

The common term a cannot be canceled. Looking at an example with numbers shows such cancellation is wrong:

$$\frac{3+2}{3+4} = \frac{5}{7} \qquad \text{while } \textbf{WRONGLY} \text{ canceling 3's gives} \qquad \frac{2}{4} = \frac{1}{2}$$

But common factors can be canceled:

$$\textbf{CORRECT!} \qquad \frac{ab}{ac} = \frac{b}{c} \qquad \textbf{CORRECT!}$$

12) **names are important**: If you are asked to solve the equation $a+2B=h$ for B, you are not free to change the names of the three variables. In particular, you can't rewrite the problem as $A+2B=H$ or $a+2b=h$, or any other variation. The names have been assigned and you are not free to change them, not even their case.

13) **the checkers speech**: In many problems it's possible to check your answer. For a simple example, if you solve the equation $2x+1=5$ and end up with $x=3$, you can check your answer by taking 2 times 3 then adding 1. The total is 7, not 5, so the answer is wrong. Of course, when an answer is wrong, the next step is to review the solution to locate and fix the error.

Sometimes you can look at an answer and see it does not make any sense. For example if the problem asks the amount of interest will be earned in 5 years if \$100 is invested at 3%, and your answer is \$2,391,234.97, obviously something is wacky. As another example, suppose the problem asks how long it will take Al and Bill to mow the lawn working together if Al can do the job alone in 4 hours while Bill needs 6 hours to do the job. If you end up with the answer that it takes 5 hours working together, then that's just plain crazy since that is longer than it takes Al working alone.

So, when possible, check your answers and check the reasonableness of your answers.

14) **mathematical magic**: Now and then I see the solution to a problem that makes absolutely no sense, and yet, as if by magic, the correct answer appears at the end of the problem. As a silly example of this sort of thing, suppose the problem reads *Al has two apples. Bill gives Al one apple. How many apples does Al have now?* Solution: In the name Bill we find **ll**. In the name Al we find **l**. Putting them together gives **lll** which is the Roman numeral three. So the answer must be 3.

Of course, these sorts of *magical* solutions are usually a bit more subtle. The point is that the work leading to a solution has to make sense. I don't give any credit for such magical solutions, despite pleas of *But I got the right answer!*

Remember, the point of the assigned problems is not just to get the correct answer to the assigned problem, it is also to learn and practice techniques that you can use in the future to solve similar problems. For example, while the magical solution above gets the right answer for the given problem, and it works OK if the two people happened to be named Sally and Ralph, it fails miserably if the two people happened to be named Lilly and Allan.

In this section I will add new errors from time to time as I spot them on assignments.

15) **$\Rightarrow$ isn't $=$**: The symbols $=$ and $\Rightarrow$ cannot be used interchangeably. When writing $=$, there has to be an expression on each side of the sign (or at least it must be clear to the reader which expressions go on each side), and the two expressions must be equal. So $3(x + 1) = 3x + 3$ is an appropriate use of $=$.

The symbol $\Rightarrow$ is shorthand for the phrase *it follows that* (or *so* or *hence* or *implies*), at least in informal mathematics which we are talking about here. (In very formal mathematics and logic, $\Rightarrow$ has a more specialized meaning which need not concern us here.) Writing $2x + 1 = 5 \Rightarrow 2x = 4$ is an appropriate use of $\Rightarrow$.

Here are two ways these symbols are misused:

1) $3(x + 1) \Rightarrow 3x + 3$ should be $3(x + 1) = 3x + 3$.

2) $2x + 1 = 5 = 2x = 4$ should be $2x + 1 = 5 \Rightarrow 2x = 4$.

16) **missplit denominators**: The following inviting looking algebra is wrong:

$$\textbf{WRONG!} \qquad \frac{1}{x^2+x^3} = x^{-2} + x^{-3} \qquad \textbf{WRONG!}$$

The error made was splitting the quotient $\frac{1}{x^2+x^3}$ into $\frac{1}{x^2} + \frac{1}{x^3}$, and then rewriting each term. But those two expressions are not the same, and so the algebra is wrong. You can see they are different by looking an example with specific numbers. When $x = 2$ the expression $\frac{1}{x^2+x^3}$ becomes $\frac{1}{2^2+2^3} = \frac{1}{4+8} = \frac{1}{12}$ but $x^{-2} + x^{-3}$ becomes $2^{-2} + 2^{-3} = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$, definitely not $\frac{1}{12}$.

17) **missed opportunities**: The equation $x^2 = 3x$ cannot be solved by canceling an x from each side to conclude $x = 3$. The problem is that it is only legal to cancel a *non-zero* common factor. The reason is that when a common factor is canceled, there is actually a division taking place, and division by 0 is not defined. The correct way to solve $x^2 = 3x$ would be to write it as $x^2 - 3x = 0$, and so $x(x-3) = 0$. That tells us either $x = 0$ or $x = 3$ and so we conclude $x = 0, 3$ are the two solutions, one of which was missed by the invalid canceling.

18) **haste makes wackiness**: Consider the following solution to the inequality $1 - 3x < 7$:

$$1 - 3x < 7$$

$$-3x < 6$$

$$\frac{-3x}{-3} > \frac{6}{-3}$$

$$x > -2$$

That is a good solution. But sometimes the second and third steps are combined into a single step and the solution looks like

WRONG!

$$1 - 3x < 7$$

$$\frac{-3x}{-3} < \frac{6}{-3}$$

$$x > -2$$

The problem is that the inequality in the second line is now backwards. The line $-3x < 6$ was written down, then instead of beginning a new line where each side is divided by -3, the division was simply pasted into the line $-3x < 6$, the result being the backwards inequality. Don't be in such a hurry. Write in the missing line. Remember that each statement you write as part of the solution has to be correct.

19) **short changed**: Be sure to write all of a square root sign, a division sign, and so on. For example, to write the square root of $3x^2 + 7y + z$, write

$$\sqrt{3x^2 + 7y + z} \qquad \text{and not} \qquad \sqrt{3x^2 + 7}y + z$$

and when writing a fraction with numerator $3x^2 + 7y + z$ and denominator w, write

$$\frac{3x^2 + 7y + z}{w} \qquad \text{and not} \qquad 3x^2 + \frac{7y}{w} + z \qquad \text{or} \qquad 3x^2 + \frac{7y}{w} + z$$

Some Slip-Ups in Calculus

20) **missing limits**: Problem: Compute $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$. Solution:

WRONG!

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{(x+1)(x-1)}{x - 1} = x + 1 = 2$$

WRONG!

The error occurs at the first step. Is isn't true that $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ equals $\frac{(x+1)(x-1)}{x - 1}$. In fact, it is the limit as $x \to 1$ of the second expression that equals the first, and without that $\lim_{x \to 1}$ in there, the equality is wrong.

A correct solution looks could look like

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x+1)(x-1)}{x - 1} = \lim_{x \to 1} (x + 1) = 2$$

21) **son of missing limits**: When computing the derivative of $f(x) = x^2$ directly from the definition, the following computation is wrong:

WRONG!

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h = 2x$$

WRONG!

The error comes at the second equality. It is not true that

WRONG!

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

WRONG!

The expression on the right needs $\lim_{h \to 0}$ in front of it for the two sides to be equal. The $\lim_{h \to 0}$ has to appear until the limit is actually taken. So a correct solution looks like this:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \to 0} (2x + h) = 2x \end{aligned}$$

22) **where's my prime**: Problem: Find the derivative of $f(x) = x^2 + 3x$. Solution:

WRONG!

$$f(x) = x^2 + 3x = 2x + 3$$

WRONG!

The error is that $x^2 + 3x$ does not equal $2x + 3$. The second expression equals the *derivative* of the first. So a correct solution looks like:

$$f(x) = x^2 + 3x \qquad \text{so} \qquad f'(x) = 2x + 3$$

Converting files to pdf format

Concerning converting to .pdf format in Windows, Mac, and Linux (assuming your scanner does not have a save as pdf option, though most seem to have that):

Assuming you are using Windows:

Go to

http://www.pdf995.com/

There you can download a free program called pdf995 which will convert any printable file to pdf format.

You can pay $10 ($9.95 actually, the reason for the name of the product) to avoid seeing a brief *sponsor ad* with each conversion, but I just use the free version and close the ad window when it opens.

This is excellent software, and a breeze to use.

Besides dozens of other operations, it can combine several pdfs into a single file which is very handy for uploading files to Blackboard.

Another freeware program that does the same sort of thing without the ad is PDFCreator available at

http://www.pdfforge.org/products/pdfcreator

If you have a little bit of money burning a hole in your pocket, for $30 you can buy a copy of BlueBeam, the ultimate pdf software that can do everything pdf995 and PDFCreator do and a whole lot more, particularly if you have a tablet PC.

The standard price for BlueBeam is $150, but it is available to people involved with education (teachers, students, etc) for $30 from

http://www.bluebeam.com/web07/us/store/education_store.asp

The order needs to be sent from an .edu address such as your und mail account.

If you have a Mac (OS X 10.5 anyhow) it's even easier:

Here's the method I use:

Open the .jpeg (or whatever) file in Preview.

Click "Save As ...". an select PDF.

To combine several pdf files into a single file:

Open the first file in Preview. Open the sidebar by going to the taskbar at the top of the screen, selecting View and then Sidebar from the menu. Drag the pages you want to combine to the sidebar. Adjust the order of the pages if necessary in the sidebar.

Now hit Save As to save the combined pdfs.

Finally, for Linux (I use Kubuntu Linux)

This is a two step process:

Most applications will save in postscript format (.ps). There is a standard program ps2pdf that will convert the .ps to a .pdf file.

There is another linux app that will combine multiple pdf files into a single file.

Install ghostscript and pdftk.

To combine file1 file2 ...

run

gs -dBATCH -dNOPAUSE -q -sDEVICE=pdfwrite -sOutputFile=finished.pdf file1.pdf file2.pdf ...

I've never tried another option:

joinPDF (Google it).

Let me know if you have any questions about these pieces of software.