

Future Directions for Research in Geometry:

A summary of the roundtable discussion held at the Twelfth Midwest Geometry Conference

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August 8, 2002

The Twelfth Midwest Geometry Conference took place on the University of North Dakota campus on April 26, 27, and 28, 2002. In addition to eleven invited talks, the conference included a special roundtable session on the state of the art, current directions, and emerging opportunities. Many of the conference participants attended this session and expressed their views. Of several exciting current directions in geometry, a few came up for extended discussion, which we summarize in this paper.

An overarching theme of the roundtable was the need to find tractable problems and entry points for Ph.D. students. It was agreed that since geometry has a rather high “entry cost” for students, it is important to choose Ph.D. problems carefully. Participants specifically noted that we face the challenge of passing the art of mathematical and scientific research on to the next generation. The “entry cost” issue is thus of critical importance to the research community.

One area the roundtable focused on was the topic of curvature flows. Since the pioneering work of Hamilton in the early 1980s, the Ricci flow and related problems have shown themselves to be very involved, but also deeply revealing of geometric structure. Understanding singularities that develop in curvature flows is likely to be the focus of much work in the coming years. Some participants mentioned the need and opportunity to study discrete analogues of curvature flows, which should bring one into contact with combinatorial differential operators (like the combinatorial Laplacian).

Discussion at the roundtable also turned to the interaction of mathematics with physics, especially differential geometric methods in mathematical physics. There are open problems related to the Cosmic Censorship Conjecture, the application of quantum topology to physics, and the AdS/CFT correspondence. Conformally invariant differential operators and differential operator invariants of other parabolic geometries (like projective and CR geometry) are expected to find important applications in physics. In connection with the Cosmic Censorship Conjecture, it was remarked that Lorentzian geometry is

quite underdeveloped relative to Riemannian geometry – this represents an opportunity for future work.

Spinors, the Dirac operator, and related topics should be important in their own right in geometry, as well as in making connections with physics. For some reason, this topic seems to be pursued only by a very small fraction of geometers, especially in the USA. (France and Germany, in contrast, have sizable groups in spin geometry.)

The theory of knots, including such concepts as links and knot energy, was mentioned as a mathematically important area in which the Midwest is fairly strong. One long-term goal is the discovery of a Gauss-Bonnet Theorem for knots. Knot theory is also important as a source of tractable problems that may be attacked by Ph.D. students. Much of knot theory involves fairly concrete objects, such as knot invariants, that one can manipulate with few prerequisites. Problems in knot theory, moreover, often involve automated computation, so that students can actually make some sort of progress immediately. Students can use computers to simulate electrically charged knots and let them flow to a minimal energy state.

These types of tie-ins with automated computation could be a way to combat the “entry cost” problem of geometry in general. Such tie-ins would include, in addition to automated numerical and graphical procedures, automated symbolic computation in (for example) research on the invariants and invariant differential operators for the parabolic geometries mentioned above. Automated computation is also a way to bring undergraduate research students into the mix, thus achieving a measure of vertical integration.

In summary, the most general conclusion to be drawn from the roundtable discussion is that geometry and the various fields related to geometry offer many challenges as well as opportunities to experienced researchers and to Ph.D. students and other new researchers as well.

Final Notes

Readers are encouraged to copy this paper and pass it on to their colleagues. For further information on the Twelfth Midwest Geometry Conference, and for an electronic copy of this paper, see the conference Web site, at

<http://www.und.nodak.edu/dept/math/mgc2002>.

This material is based upon work supported by the National Science Foundation under Grant No. DMS-0202812, and by North Dakota State University, North Dakota EPSCoR (through NSF grant # EPS-9874802), and the University of North Dakota. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation or the other sponsors.