Math 208, Handout I: A Few Optional Problems

(1) Let $p(x, y)$ be the statement "x has defeated y." Let the domain of discourse be the set of all major league baseball teams. Use symbols and quantifiers to express each of the following statements:

(a) No team has defeated every team.

(b) There is a team that has not defeated any team.

(c) Every team has defeated some team.

(d) Every team has been defeated at least once.

(e) There is a team that has defeated every team.

(f) There is a team that has been defeated by every team.

(g) No team has defeated itself.

(h) There is a team that has defeated every other team.

(2) Negate each of the symbolic statements that you constructed in Exercise 1.

(3) Let $p(x, y)$ be as above. Translate each of the following statements into English:

(a) $\forall\, x\, \exists\, y\ p(x, y)$

(b) $\exists\, y\, \forall\, x\ p(x, y)$

(c) $\forall\, y\, \exists\, x\ p(x, y)$

(d) $\exists\, x\, \forall\, y\ p(x, y)$

(4) Prove each of the following logical equivalences. Do so by using the method illustrated at the end of Chapter 2.

(a) $\neg r \to \mathbb{F} \equiv r$

(b) $(p \vee \neg q) \wedge q \equiv p \wedge q$

(c) $(p \to q) \wedge r \equiv (r \to p) \to (q \wedge r)$