Math 107, Handout V: Review Problems for the Final Exam

To prepare for the exam, you may wish to study the problems in this handout, the assigned homework problems, the book, your lecture notes, and the problems in all of the previous review sheets. You may *not* use calculators during the exam.

1. Sketch the graph of each parabola, showing the vertex, focal point, and directrix.

   (a) $y = 3x^2 - 12x + 17$ (b) $y = -2x^2 - 12x - 19$ (c) $x = \frac{1}{2}y^2$

2. Determine the equation of the parabola that satisfies the given conditions.

   (a) Focus at $(2, 0)$, directrix $y = -2$.
   (b) Focus at $(2, 0)$, directrix $x = -2$.
   (c) Focus at $(3, 1)$, directrix $y = 3$.
   (d) Vertex at $(0, 0)$, directrix $y = -1$.
   (e) Vertex at $(-2, 2)$, directrix $y = -4$.
   (f) Vertex at $(1, 3)$, focus at $(1, -1)$.

3. Sketch the graph of each ellipse. Find the vertices and focal points.

   (a) $\frac{x^2}{16} + \frac{y^2}{25} = 1$ (c) $9y^2 + 4x^2 + 36y - 24x + 36 = 0$
   (b) $x^2 + \frac{y^2}{4} = 1$ (d) $4y^2 + 9x^2 - 16y + 18x - 11 = 0$

4. Determine the equation of the ellipse that satisfies the given conditions.

   (a) x-intercepts at $(\pm 7, 0)$ and y-intercepts at $(0, \pm 3)$.
   (b) Foci at $(8, 0)$ and $(-8, 0)$, and one vertex at $(10, 0)$.
   (c) Foci at $(2, -1)$ and $(2, 3)$ and major axis of length 6.
   (d) Vertices at $(-1, 3)$ and $(9, 3)$ and a focus at $(7, 3)$.

5. Sketch the graph of each hyperbola. Find the vertices, focal points, and the equations of the asymptotes.

   (a) $\frac{x^2}{36} - \frac{y^2}{25} = 1$ (c) $16x^2 - 9y^2 + 96x + 18y - 9 = 0$
   (b) $\frac{y^2}{25} - \frac{x^2}{49} = 1$ (d) $25y^2 - 9x^2 + 200y + 90x - 50 = 0$

6. Determine the equation of the hyperbola that satisfies the given conditions.

   (a) Foci at $(0, \pm 5)$ and a y-intercept at $(0, 4)$.
   (b) Vertices at $(2, 1)$ and $(8, 1)$ and a focus at $(0, 1)$.
   (c) Vertices at $(3, -2)$ and $(3, 4)$ and a focus at $(3, 6)$.
   (d) Foci at $(-4, 1)$ and $(6, 1)$ and a vertex at $(4, 1)$.

7. Convert the equation $x^2 + y^2 = 7y$ to a polar equation.

8. Convert the equation $r = 8\cos\theta$ to a rectangular equation.

9. Sketch the graph of each polar equation.

   (a) $r = 2 + 4\sin\theta$ (b) $r = 4\cos\theta$ (c) $r = 4 + 4\sin\theta$ (d) $r = 5$

10. Find a corresponding rectangular equation. Sketch the graph and show the point for which $t = 0$.

    (a) $x = 2t + 4, \quad y = t - 3, \quad -1 \le t \le 3$ (b) $x = \cos t + 1, \quad y = 2 - \sin^2 t$

11. Find parametric equations in the parameter t, where $0 \le t \le 2\pi$, for the unit circle traced three times around counterclockwise.