Math 512-513, Handout XIII: Exercises Involving Hilbert Spaces

(1) Let H be any Hilbert space. For all $x \in H$, let $\|x\| = \sqrt{(x,x)}$. Prove that $\|\cdot\|$ is a norm on H. Hints: Prove that $\|\cdot\|$ satisfies the properties in the definition of a norm given at the bottom of page 217. To prove the triangle inequality, use the Cauchy-Scwarz inequality:

$$|(x,y)| \leq \|x\| \cdot \|y\|.$$

(2) Do Exercise 50 in Section 10.8. Hint: Use the Cauchy-Schwarz inequality.

(3) Prove that $\mathbb{R}^3$ is complete with the norm

$$\|\langle x,y,z\rangle\| = \sqrt{x^2+y^2+z^2}.$$