Math 512-513, Handout I: Supplementary Information

Proposition 1: Let f and g be nonnegative measurable functions defined on a measurable set E. If $f = g$ a.e., then

$$\int_E f = \int_E g.$$

Proposition 2: Let f be a nonnegative measurable function defined on a measurable set E. Let A and B be measurable subsets of E. Suppose that f vanishes off of A and that f also vanishes off of B. Then

$$\int_A f = \int_B f.$$

Notation: Let f be a nonnegative measurable function defined on a measurable set E, and suppose f vanishes off of a measurable set $A \subset E$. We let $\int f$ denote $\int_A f$.

Proposition 3: Let f be a nonnegative measurable function defined on a measurable set E. Let a measurable set $A \subset E$ be given. Then

$$\int_A f = \int f\chi_A.$$

Proposition 4: Let f be a nonnegative measurable function defined on a measurable set E. Let A and B be measurable subsets of E, and suppose $A \cap B = \emptyset$. Then

$$\int_{A \cup B} f = \int_A f + \int_B f.$$

Proposition 5: Let f be a nonnegative measurable function defined on a measurable set E. Let A be a measurable subset of E. If $mA = 0$, then

$$\int_A f = 0.$$

Proposition 6: Let f be a nonnegative measurable function defined on a measurable set E. Let A and B be measurable subsets of E, and suppose $A \subset B$. If $m(B \sim A) = 0$, then

$$\int_A f = \int_B f.$$

Proposition 7: Let E be a measurable subset of $\mathbf{R}$. Then $\int \chi_E = mE$. This is true whether mE is finite or infinite.

Proposition 8: Let $\langle a_n \rangle_{n=1}^{\infty}$ be a sequence of extended real numbers. Suppose that ℓ is an extended real number and that $\overline{\lim}\, a_n = \underline{\lim}\, a_n = \ell$. Then $\lim a_n = \ell$.

Proposition 9: Let f be a nonnegative measurable function on a measurable set E. Then

$$\int_E f < \infty \quad \Rightarrow \quad m\{x : f(x) = \infty\} = 0.$$