Math 409: Review Problems for Final Exam (Peterson)

The final exam will be comprehensive with an emphasis on the material we have discussed since the second midterm exam. The second midterm exam occurred at about the time at which we studied Clairaut's axiom. Some material may appear on the final exam even though it does not appear on this review sheet or on either of the other previous review sheets.

1. Suppose Clairaut's Axiom holds. Prove that the sum of the measures of the four angles of a convex quadrilateral is 360°.

2. Prove that Hilbert's parallel postulate implies Clairaut's axiom. Hint: Your proof should be very short.

3. Euclid's fifth postulate is stated on page 128 of our text. Prove that Euclid's fifth postulate implies Wallis' postulate.

4. Briefly describe the Beltrami-Klein model of hyperbolic geometry.

5. In hyperbolic geometry, let a line l and a point P not on l be given. Briefly describe the left and right limiting parallel rays to l through P as well as their basic properties.

6. In hyperbolic geometry, suppose that $l||m$ and that m does not contain a limiting parallel ray to l. Prove that there exist distinct points on l which are equidistant from m.

7. Assume that the hyperbolic axiom holds. Assume that parallel lines l and l' have a common perpendicular $\overleftrightarrow{\mathrm{PQ}}$. For any point X on l, let X′ be the foot of the perpendicular from X to l'. Prove that as X recedes endlessly from P on l, the segment XX′ increases indefinitely. Hint: This is Exercise 10 in Chapter 6. The book gives some hints on how to do this exercise.

8. Assume that the hyperbolic axiom holds. Prove that the segment joining the midpoints of the base and summit of a Saccheri quadrilateral is perpendicular to both the base and the summit. Also prove that this segment is shorter than the sides of the Saccheri quadrilateral.

9. In neutral geometry, suppose $l||m$, and let distinct points P, Q, and R be given. Suppose that (1) R lies on l, (2) P and R are on opposite sides of m, and (3) Q and R are on opposite sides of m. Prove that P and Q are on the same side of l.

10. Let □ABDC be a Saccheri quadrilateral, so that ∢B and ∢A are right angles and CA ≅ DB. Prove that ∢C ≅ ∢D.

11. In hyperbolic geometry, suppose $l||m$, and suppose there exist a line t which is perpendicular to both l and m. Prove that there exists a number $\epsilon > 0$ such that for all points A on l and all points B on m, $\overline{\mathrm{AB}} > \epsilon$.

12. Suppose that the Hyperbolic Axiom holds. Thus there exist a line l and a point P not on l such that at least two lines passing through P are parallel to l. Prove that for every line m and every point Q not on m, there exist infinitely many lines passing through Q which are parallel to m.

13. In hyperbolic geometry, suppose that lines l and l' have a common perpendicular segment MM′. Prove that if A and B are any points on l such that M is the midpoint of segment AB, then A and B are equidistant from l'.