

Math 112, Handout 15: Important Trigonometric Identities

Here are some important trigonometric identities:

- (1) $\cos^2 \theta + \sin^2 \theta = 1$
- (2) $\cos(-\theta) = \cos \theta$
- (3) $\sin(-\theta) = -\sin \theta$
- (4) $\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$
- (5) $\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$
- (6) $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$
- (7) $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$

You should try to memorize the above identities. I *may* supply you with some of these identities on the final exam, but you should still try to memorize them. In identity (6), above, it may be helpful to note that if $\theta = 0$, then both sides of the identity are equal to 1. In identity (7), if $\theta = 0$, then both sides of the identity are equal to 0.

The following identities are also important. You can easily derive them from the above identities.

- (8) $\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$
- (9) $\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$
- (10) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
- (11) $\sin 2\theta = 2 \sin \theta \cos \theta$
- (12) $\cos(\theta + \pi) = -\cos \theta$
- (13) $\sin(\theta + \pi) = -\sin \theta$

Identity (8) follows from identities (2), (3), and (4), and Identity (9) follows from identities (2), (3), and (5). Identity (10) follows from (4), and Identity (11) follows from (5). Identities (12) and (13) follow from consideration of the unit circle.